Detector Modeling Techniques for Pre-Clinical 3D PET Reconstruction on the GPU


Abstract—In this paper, we present different approaches to the efficient handling of gamma interaction within the detector crystal matrix for ML-EM-based 3D reconstruction algorithms for positron emission tomography (PET). Geometric calculations as well as approximation of inter-crystal scattering in the forward and in the back-projection are calculated on-the-fly by Monte Carlo techniques implemented on the GPU. Instead of an accurate treatment of scattering simulation, the main characteristics of the scattering effect are captured by approximate models or by performing most of the calculation as a pre-processing step based on prior Monte Carlo simulations. The discussed methods involve the modification of the detector radius in the geometric model; taking only the attenuation effect into account; applying 4D filtering on the lines-of-responsive regarding the pre-calculated crystal-transport probability distributions; or replacing the simulated detector response function by an approximated geometric model; taking only the attenuation effect into account; or by performing most of the calculation as a pre-processing step based on prior Monte Carlo simulations. The discussed methods involve the modification of the detector radius in the geometric model; taking only the attenuation effect into account; applying 4D filtering on the lines-of-responsive regarding the pre-calculated crystal-transport probability distributions; or replacing the simulated detector response function by an approximated geometric model; taking only the attenuation effect into account; or by performing most of the calculation as a pre-processing step based on prior Monte Carlo simulations.

Index Terms—gamma photon scattering, GPU, positron emission tomography, reconstruction algorithms

I. INTRODUCTION

In positron emission tomography (PET) the aim is to find the spatial intensity distribution of positron-electron annihilations. During an annihilation event, two oppositely directed photons are produced, which may be absorbed or scattered both in the medium and in the detector crystals [1], [2]. The input of the reconstruction algorithm is a set of measured number of simultaneous photon incidents in these crystal pairs, aka lines of responses (LORs). The output of the reconstruction is the emission density function \( x(\vec{v}) \) that describes the number of events annihilated in a unit volume per second around point \( \vec{v} \).

In iterative reconstruction schemes, forward projections and corrective back projections alternate. The forward projection simulates the particle transport and computes the expected responses from the current estimation of the emission density function [3], [4]. The correspondence between the emission density function \( x(\vec{v}) \) and the detector responses is built of the elemental conditional probability density that a photon pair is detected by the two crystals forming LOR \( L \) given that they are emitted at point \( \vec{v} \) in directions \( \vec{\omega} \) and \(-\vec{\omega}\), which is denoted by \( P(\vec{v}, \vec{\omega} \rightarrow L) \).

If a photon pair is isotropically emitted from point \( \vec{v} \), then the expectation value of the photon incidents in LOR \( L \) is:
\[
\tilde{y}_L = \int \int x(\vec{v}) P(\vec{v}, \vec{\omega} \rightarrow L) \frac{d\omega}{2\pi} d\nu
\]
where \( V \) is the volume of interest and \( \Omega_H \) is the directional set of a hemisphere.

A simplified, geometry-only reconstruction model assumes that the detectors always absorb any photon that arrives at their surface. With this assumption, the expected number of hits in a LOR connecting detector crystals \( d_1 \) and \( d_2 \) is:
\[
\tilde{y}_{geom}(d_1, d_2) = \int \int G(\tilde{z}_1, \tilde{z}_2) X(\tilde{z}_1, \tilde{z}_2) d\tilde{z}_2 d\tilde{z}_1
\]
where \( D_1 \) and \( D_2 \) are the surfaces of the two detectors,
\[
G(\tilde{z}_1, \tilde{z}_2) = \frac{\cos \theta_{\tilde{z}_1} \cos \theta_{\tilde{z}_2}}{2\pi|\tilde{z}_1 - \tilde{z}_2|^2}
\]
is the geometry factor expressed by \( \theta_{\tilde{z}_1} \) and \( \theta_{\tilde{z}_2} \) that are the angles between the surface normals and the line connecting points \( \tilde{z}_1 \) and \( \tilde{z}_2 \) on the two detector surfaces, and
\[
X(\tilde{z}_1, \tilde{z}_2) = \int_{\tilde{z}_1}^{\tilde{z}_2} x(\vec{l}) d\vec{l}
\]
is a line integral of emission density \( x(\vec{l}) \) between endpoints \( \tilde{z}_1 \) and \( \tilde{z}_2 \).

However, photons may be scattered in the detector crystals before they are finally absorbed (see Fig. 1), which makes the simplified, geometric reconstruction approach inaccurate [1]. Accurate reconstructions need the consideration of crystal scattering and absorption phenomena. However, on-line full Monte Carlo simulation of detector scattering would be too
time consuming. On the other hand, neither the full pre-computation of this photon transport data is feasible since when scattering is also considered, the system matrix is not sparse, so its storage requirements are prohibitive [5].

Therefore, in this paper we propose techniques that factorize the problem and we use pre-computation only for a single factor that represents detector scattering. In the following sections we propose approaches of different compromises between pre-computation and on-line simulation. By presenting our results, we also show that these methods can be efficiently implemented on a GPU.

II. METHODS

A. “Effective Radius” Model

Our first approximate detector response model is the introduction of an effective detector ring radius. The model is very easy to implement, as it needs only the modification of a single parameter of the geometric model. The \( z_1, z_2 \) interaction points of the geometry-only model are shifted radially outwards by a constant displacement. This “inflation” of the detector ring causes an effective correction of LOR directions without the need for memory and time consuming inter-crystal displacement of LOR endpoints.

The difference of the real and effective radii is chosen to be equal to the average depth of gamma interaction and it is calculated from Monte Carlo simulation of the detector module [6]. For the NanoPET\(^{TM}\)/CT scanner with LYSO crystals with the depth of 13 mm, radius increase is found to be around 5–6 mm [7]. Interestingly, the visual quality of the reconstructed images is rather insensitive to the precise value of the effective radius. Although the average depth of interaction slightly varies with the incidence angle of the incoming photon, we can neglect the incident angle for the ease of computation, since the average depth is almost constant below 50 degrees.

B. Attenuation-Only Model

A more accurate model can be defined to approximate the detector response by modeling the absorption within the detector crystals whereas still neglecting the accurate modeling of the inter-crystal scattering. In this model, LOR endpoints are chosen from inside the detector crystals. Photons can either be detected in the detector whose surface is crossed or they can fly through the detector without getting absorbed. In this way, this technique also models the phenomenon that a crystal does not necessarily absorb the incident photon.

With this assumption, the geometric formulation resulted in the following expected number of hits in a LOR connecting detector crystals \(d_1\) and \(d_2\):

\[
\bar{y}_{det}^{att}(d_1,d_2) = \int_{V_1} \int_{V_2} G'(z_1, z_2) X(z_1, z_2) p_{att}(z_1, \omega_{z_1 \rightarrow z_2}) \cdot p_{att}(z_2, \omega_{z_2 \rightarrow z_1}) dz_2 dz_1,
\]

where \(V_1\) and \(V_2\) are the volumes of the two detectors,

\[
G'(z_1, z_2) = \frac{1}{2\pi |z_1 - z_2|^2}
\]

is the volumetric geometry factor and

\[
p_{att}(z_1, \omega) = \sigma_{att} e^{-\sigma_{att} l}
\]

is the absorption probability density function specifying the probability density that the photon coming from direction \(\omega\) is absorbed in point \(z_1\). If scattering is ignored in the measured medium, which is a good estimation for pre-clinical systems, all incident photons have the same energy [3], [7]. This enables us to express the probability density function by the absorption coefficient \(\sigma_{att}\) and length \(l\) of the route that the photon travels inside the crystal.

C. 4D LOR Filtering

Besides the attenuation, photons may also get scattered in the crystal matrix before they get finally absorbed [2]. Scattering in the detector can be modeled by a transport function as \(E_i(z, \omega, \rightarrow d)\) that gives the expected number of hits reported in crystal \(d\) provided that a photon enters the detector module at point \(z\) from a direction \(\omega\). As for the previous model, a constant energy for the incident photons is assumed. Incorporating the transfer function into the expected value formula, we get the following expected LOR value model:

\[
\bar{y}_{det}^{4D}(d_1,d_2) = \int_{D} \int_{D} G(z_1, z_2) X(z_1, z_2) E_i(z_1, \omega \rightarrow d_1) \cdot E_i(z_2, \omega \rightarrow d_2) dz_2 dz_1,
\]

where \(\omega\) is the direction vector pointing from \(z_1\) to \(z_2\).

Theoretically, one should integrate over the total surface \(D\) of all detector modules. However in practice, only the closer neighborhood of a scintillating crystal is important. The transport function is defined for points and directions, and it needs to be represented by finite data. In order to avoid the high memory requirement of sophisticated finite-element techniques, we apply a simpler discrete, piecewise constant approximate scheme and we also factorize the model and store only averages for different surface points per detector crystal:

\[
\bar{y}_{det}^{4D}(d_1,d_2) = \sum_{j} \sum_{i} \bar{y}_{L,(i,j)}^{geom} \cdot p_{i \rightarrow d_1}(\phi_{i,j}, \theta_{i,j}) \cdot p_{j \rightarrow d_2}(\phi_{i,j}, \theta_{i,j})
\]

where \(p_{i \rightarrow d_1}(\omega)\) is the crystal transport probability that represents the photon transport between the crystals. The final system response is the convolution of the geometric response...
and the crystal transport probability matrix. Since a LOR involves two detector crystals, this results in a four-dimensional convolution. By organizing the calculations by their outputs, 4D LOR filtering fits well to the architecture of the GPUs [8], [9].

For the NanoPET(TM)/CT scanner, the transport function $E_t$ has been computed by GATE [6] as a pre-processing step. Incident photons arriving from a direction of given inclination and azimuth angles at uniformly distributed points on the detector surface are simulated and the discrete crystal transport probability function $p_{1 \rightarrow d}(\vec{x})$ is calculated. This pre-calculation process takes a very short time compared to the calculation of each system matrix element for a given PET system [5].

### D. Parametrized Model

By using a parametrized model, a different approximation of Eq. (1) can be defined that can fit better to the photon transport-based Monte Carlo estimators [10], [11]. This approach utilizes that $E_t$ can be approximated by a combination of simple continuous functions. Exploiting the radial symmetries of the transport function with respect to $\vec{\omega}$, $E_t$ can be approximated as a separable extension of a function along the flight path of the particle and a function defined in the perpendicular plane:

$$\tilde{y}_{L}(d_1,d_2) = \int_{D_{1}} \int_{D_{2}} p^{||}(\vec{z}_1,\vec{\omega} \rightarrow \vec{d}_1)p_{\perp}(\vec{z}_1,\vec{\omega} \rightarrow \vec{d}_1)d_1d_2,$$

where $p^{||}$ and $p_{\perp}$ have been approximated in the following form:

$$p^{||}(d) = \left(1 - \frac{1}{2 + 2d}\right) \cdot (0.00604 \cdot e^{\frac{d}{0.52}} - 8.221^{-5}) \text{[mm]}$$

$$p_{\perp}(r) = \frac{1}{0.6 \sqrt{2\pi}} e^{-\frac{r^2}{2}} \text{[mm]}.$$

which are calculated by fitting a parametrized model to the pre-computed probability distributions already calculated for the 4D filtering method.

### III. RESULTS

For demonstrating the performance of the proposed methods, we have modeled the geometry and crystal matrix of the NanoPET(TM)/CT pre-clinical imaging system. In case of this PET scanner, 12 detector modules are organized in a ring, with each detector module consisting of 81 x 39 tightly packed LYSO crystals (1.12 mm x 1.12 mm x 13 mm). Considering 1 to 3 coincidence relations, the total number of LORs is $18 \cdot (81 \times 39)^2 \approx 180$ million [7].

The proposed methods have been implemented in CUDA and have been executed on an Nvidia GeForce 480 GTX GPU [12]. Due to the massively parallel hardware of the GPU, even the most sophisticated 4D LOR filtering requires only 5 seconds, which is negligible with respect to the time of geometric LOR computation. Thus, our proposed detector modeling techniques have practically no overhead.

The reconstruction algorithm that we apply is an iterative maximum likelihood estimation method (ML-EM), which alternately executes photon transport simulation (forward projection) and source correction (back projection). We have implemented the forward and back projectors on the GPU in two different ways: by using Monte Carlo particle transport simulation and by using adjoint Monte Carlo approximation [13].

The “effective radius” and the attenuation-only models have been incorporated into both methods, whereas 4D LOR filtering has been especially designed for the adjoint approach and the parametrized scattering model has been designed for the particle transport Monte Carlo method.

The verification of the 3D reconstruction implementations was performed by using mathematical phantoms, such as the micro Denzio phantom, with rod sizes varying from 1.0 mm to 1.5 mm, and then SNR phantom which is a cylinder filled with homogeneous activity as a background containing a hot and a cold rod.

Preliminary results for the reconstruction of the Denzio phantom are illustrated in Fig. 3, where the same slices are depicted for different detector modeling approaches using the adjoint MC engine applied for the reconstruction. Fig. 4 illustrates the L2 and a correlation-based (CC) error for the “effective radius” model and for the 4D LOR filtering approach. The CC error is invariant to the linear transformation of the activity distribution, therefore it measures the reconstruction of the original activity distribution in a qualitative way, whereas the L2 error measures the error of reconstruction quantitatively. Fig. 5 illustrates transaxial, coronal, and sagittal slices of the reconstructed volume for the signal-to-noise ratio (SNR) phantom. The results with the “effective radius” model is illustrated in the upper row, whereas the results with the 4D LOR filtering model is presented in the second row. Note that when the detector model of reconstruction is more consistent with the simulation, the resulted volume contains less noise.

Further detailed discussion including all the upper described models will be presented, including also reconstruction results of real phantom measurements and small animal acquisitions taken on a NanoPET(TM)/CT system.
Fig. 4. Correlation-based and L2 error curves of the simulated Derenzo phantom in the function of the number of iterations in case of the geometric-only, the “effective radius”, and the 4D LOR filtering reconstruction schemes.

Fig. 5. Reconstruction of the SNR phantom, simulated using the realistic detector model of GATE. The SNR phantom consists of a homogeneous cylinder of low activity, having a homogeneous hot rod and a homogeneous cold rod inside. Using “effective radius” model during the reconstruction results in a noisy image (upper row), whereas the 4D LOR filtering model (lower row) greatly reduces noise level and increases homogeneity.

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