Efficient Free Path Sampling in Inhomogeneous Media

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Abstract

This paper presents an efficient algorithm to sample free path in inhomogeneous participating media. The method is based on the concept of mixing "virtual" particles to the medium, that complete the extinction coefficient to a piece-wise constant function. We use the sampling formulae developed for piece-wise homogeneous medium, but decide randomly whether a real or a virtual particle is hit.

1. Introduction

Participating media global illumination algorithms typically use Monte Carlo quadrature and trace photons randomly in the medium [JC98]. Generating a single step of the random path involves the sampling of the free path made by the photon before scattering, deciding whether absorption happens, and finally sampling the new direction. This paper deals with the sampling of the free path.

Importance sampling requires the placement of discrete samples proportionally to the integrand, which is an exponential decay for the unscattered light. Sampling proportionally to a prescribed function can be done by the *inversion method*. The inversion method first calculates the probability density as the normalization of the original function, then obtains the desired *cumulative probability distribution* (*CDF*) as the integral of the probability density, and finally generates the discrete samples by inverting the CDF for values that are uniformly distributed in the unit interval.

The CDF of the free path length is

$$P(s) = 1 - \exp\left(-\int_0^s \sigma_t(\tau)\mathrm{d}\tau\right)$$

where s and τ are the ray parameters describing the distance from the last scattering point along the ray, and $\sigma_t(\tau)$ is the *extinction coefficient*. Thus, the free path length sample s corresponding to a uniform random number r is the solution of the following equation:

$$r = P(s) \quad \Leftrightarrow \quad -\log(1-r) = \int_0^s \sigma_t(\tau)\mathrm{d}\tau.$$

When the medium is inhomogeneous, the extinction coefficient is not constant but is represented by a voxel grid. In this case, the usual approach is *ray marching* that takes small constant steps Δs along the ray and checks when CDF $P(n\Delta s)$ gets larger than r:

$$\sum_{i=0}^{n-1} \sigma_t(i\Delta s)\Delta s \leq -\log(1-r) < \sum_{i=0}^{n} \sigma_t(i\Delta s)\Delta s.$$

Unfortunately, this algorithm requires a lot of voxel array fetches, especially when the voxel array is large and the average extinction coefficients are small.

Woodcock tracking [WMHL65] provides an alternative method to *ray marching* and samples the free path length with the maximal extinction coefficient $\sigma_{\max}$, advances the ray with sampled distance $-(\log(1-r))/\sigma_{\max}$, and accepts/rejects the point with probability $\sigma_t/\sigma_{\max}$ and $1 - \sigma_t/\sigma_{\max}$. In case of rejected scattering, and the same sampling step is repeated from there. Although Woodcock tracking offers resolution independent free path sampling, it becomes inefficient when maximum extinction $\sigma_{\max}$ is much greater than the actual extinction in a region. Our proposed method relates to Woodcock tracking but is more general and does not suffer from this problem.

2. The method of virtual particles

The extinction coefficient is the product of the the cross section of the particles forming the medium and their density. Based on this recognition, the problem of heterogeneous volumes can be handled if we mix additional, so called *virtual particles* into the medium to make the resulting extinction coefficient function easier to sample $\sigma(s) = \sigma_t(s) + \sigma_v(s)$.

However, these virtual particles should not alter the radiance inside the medium, that is, they should not change the energy and the direction of photons during scattering. This requirement is met if the virtual particle has *albedo* 1, and its *phase function* is a Dirac-delta, i.e. it alters neither the photon's energy nor its direction with probability 1.

During scattering we have to determine whether it happened on a real or on a virtual particle, so we need to use the albedo and the phase function of the original material or the parameters of the virtual particle. As sampling is required to generate random points with a prescribed probability density, it is enough to solve this problem randomly with the proper probabilities. As the extinction parameters define the probability density of scattering, their ratio gives us the probability whether scattering happened on a real or on a virtual particle. Summarizing, the free path sampling process is:

1. Generate a sample using the combined extinction coefficient $\sigma(s)$.
2. Decide randomly with probability $\sigma_t(s)/\sigma(s)$ whether a real scattering happened. If this is the case, the new scattering direction is sampled with the albedo and the phase function of the original material.
3. If virtual scattering happened, then the particle's direction is not altered and a similar sampling step is repeated from the scattering point.

Note that this algorithm is correct with any non-negative virtual scattering coefficient $\sigma_v(s)$, which opens a lot of possibilities for developing various strategies. Here we consider just a single strategy that decomposes the volume to a low-resolution grid of macrocells and assigns to macrocell i the maximum extinction coefficient $\sigma^{(i)}_{\max}$ of the voxels inside it. This implicitly defines virtual particle density as $\sigma_v(s) = \sigma^{(i)}_{\max} - \sigma_t(s)$. We execute a 3D DDA like traversal [FTK86] in the macrocells and find the macrocell that contains the scattering point. The ray parameters of the intersections with the macrocell boundaries are denoted by $s_0, \ldots, s_n$. The inequalities selecting the marcocell containing the scattering point are:

$$\sum_{i=0}^{n-1} \sigma^{(i)}_{\max} \Delta s_i \leq -\log(1-r) < \sum_{i=0}^{n} \sigma^{(i)}_{\max} \Delta s_i.$$

Unlike in ray marching, here steps $\Delta s_i = s_{i+1} - s_i$ are not constant but are obtained as the length of the intersection of the ray and the macrocell. When the inequalities are first satisfied, the macrocell of the scattering point is located. The actual scattering point is computed as

$$\sigma^{(n)}_{\max} \Delta s = -\log(1-r) - \sum_{i=0}^{n-1} \sigma^{(i)}_{\max} \Delta s_i \quad \Leftrightarrow$$

$$s = s_n + \Delta s = s_n - \frac{\log(1-r) + \sum_{i=0}^{n-1} \sigma^{(i)}_{\max} \Delta s_i}{\sigma^{(n)}_{\max}}.$$

3. Results and conclusions

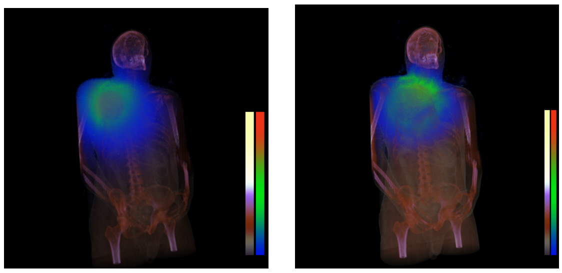

Figure 1: *Interactive volumetric global illumination tracing 25 million photon paths up to 5 scattering events where the free path is sampled by the proposed method. The two images show the visible human when the source is placed at two different positions. The bars depict the transfer functions of the density and the radiance.*

The proposed methods have been implemented in CUDA as a part of a photon mapping renderer (Figure 1) and a radiotherapy treatment design application. By shrinking the macrocells, the probability of hitting a virtual particle can be reduced at the additional cost if visiting more macrocells. When rendering the visible human of resolution 512^3, ray marching identified the next scattering point with about 100 texture fetches in average, while the proposed method needed 1.3 texture fetches of the high resolution volume in average and 3.2 texture fetches of the macrocell volume when the macrocell contained 32^3 voxels. So the macrocell size offers a flexible compromise, which should be selected according to the variation and the average of the extinction coefficient function. Our future work will aim at adaptively defining the macrocells and the incorporating non piece-wise constant upper-bounds.

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